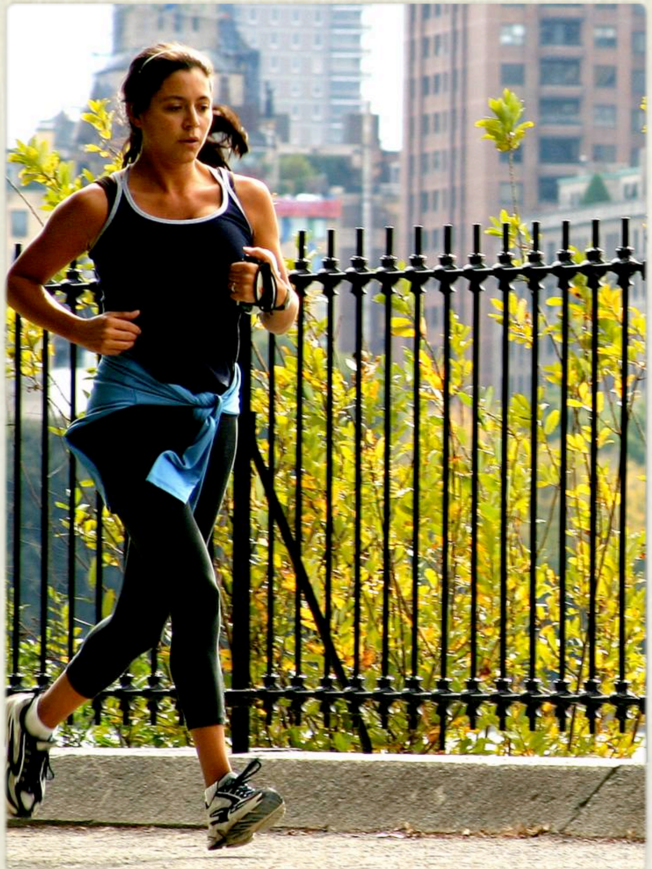


APS MULTILEVEL MODELING WORKSHOP

Elizabeth Page-Gould

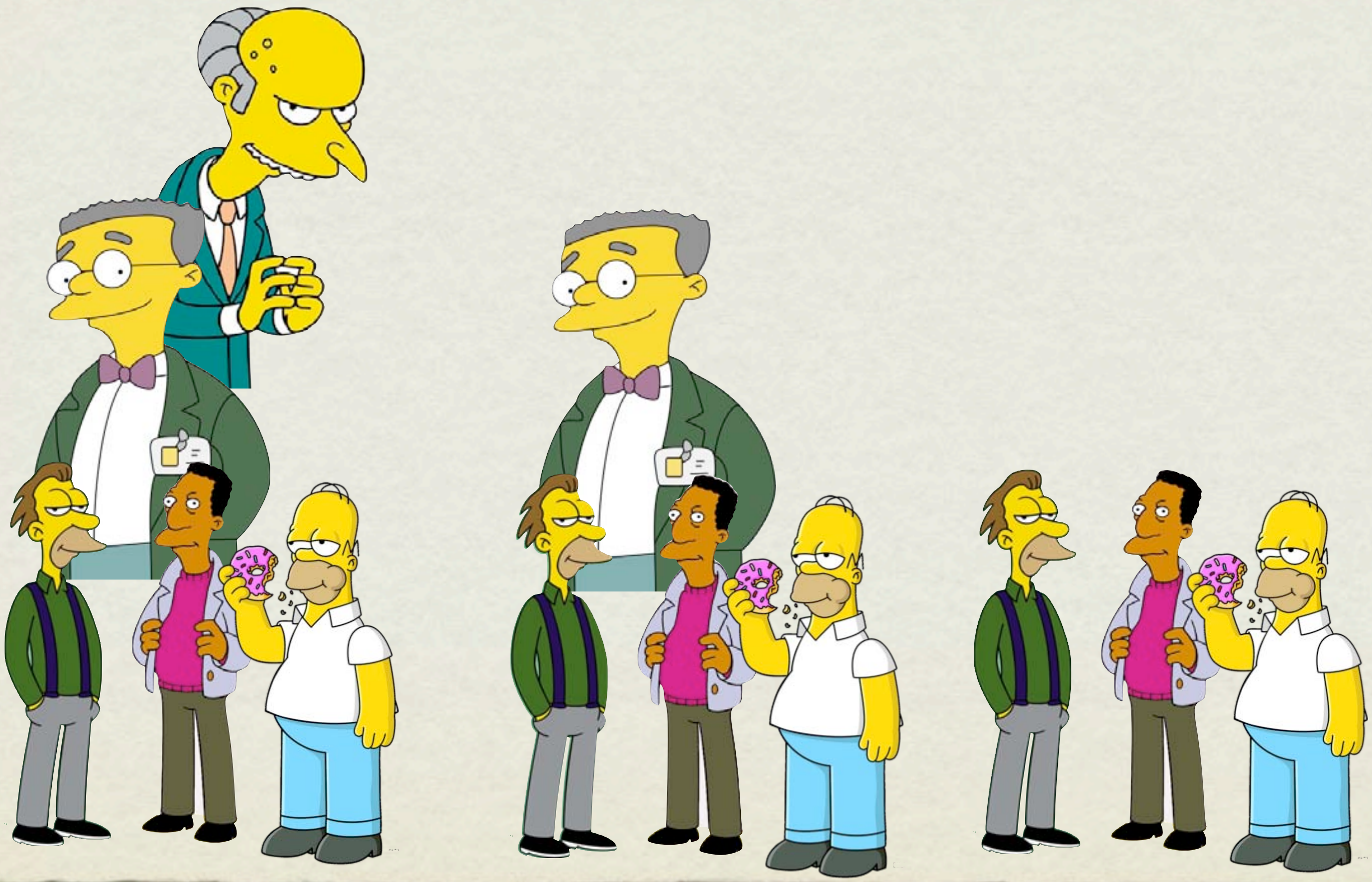
University of Toronto

WORKSHOP SPONSORS

- Association for Psychological Science (APS)
- Society of Multivariate Experimental Psychology (SMEP)

MULTILEVEL MODELING

- *A broad class of analyses that deal with hierarchy in your data*

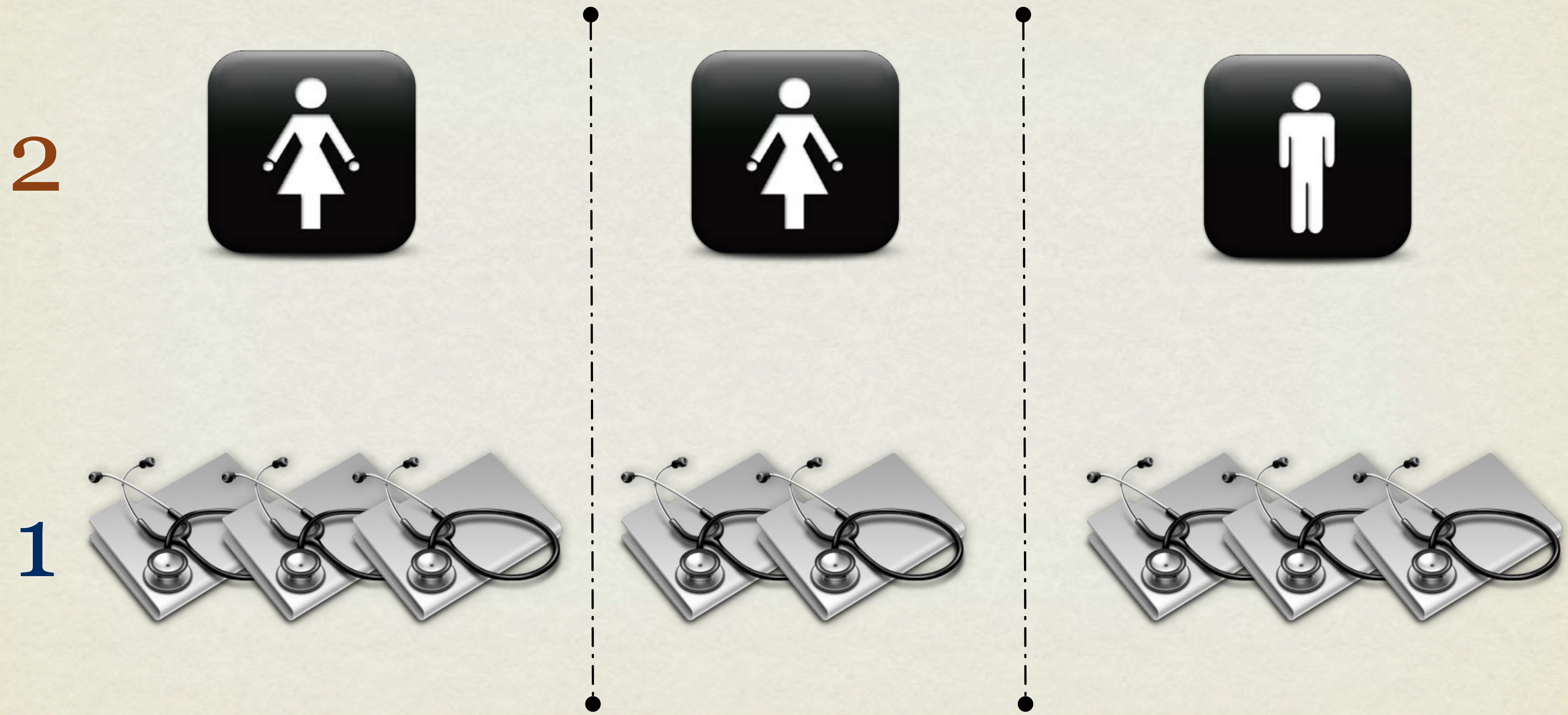


HIERARCHICAL DATA

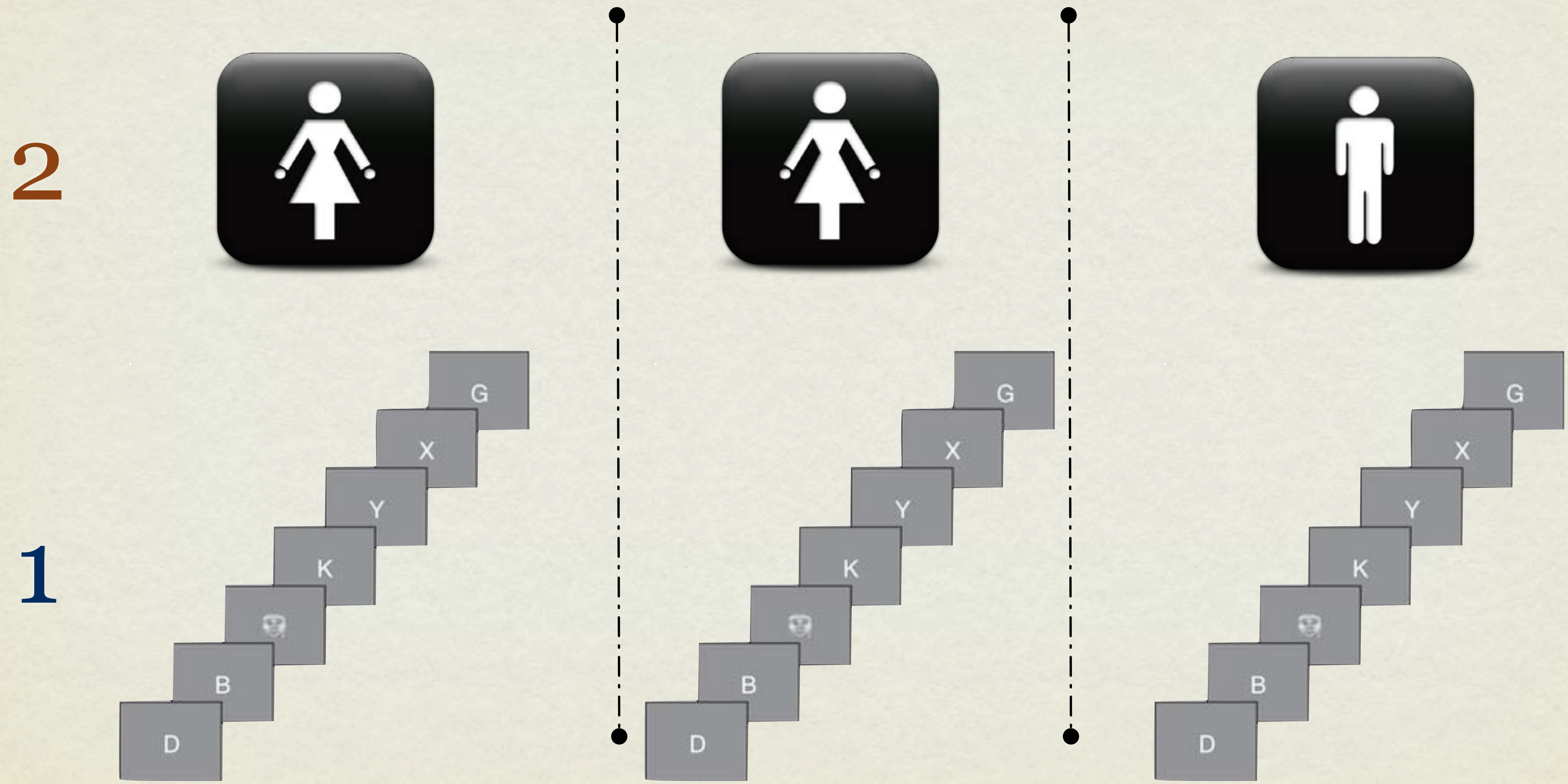
2

1

HIERARCHICAL DATA



HIERARCHICAL DATA



MULTILEVEL MODELING

- Broad class of techniques
- 1 face, many names:
  - Hierarchical linear modeling (HLM)
  - Mixed models
  - Random effects models
  - Random coefficient models
  - Growth curves & nested growth curves
  - Covariance components models

WORKSHOP OVERVIEW

- Example dataset
- Conceptual background
- Conducting the analysis
  - 2-level models
  - Best practices
  - Effect size, sample size, power
- Advanced MLM
  - *N*-level models
  - Nested growth curves
  - Cross-classified models
  - Multilevel mediation

NATIONAL LONGITUDINAL SURVEY OF YOUTH 1997

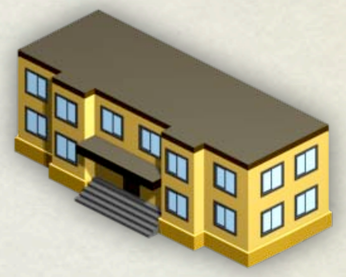
U.S. Bureau of Labor Statistics

NLSY97

- Research question
  - What factors contribute to the well-being, career success, and criminality of young Americans?
- Method
  - Interview ~9000 adolescents and young adults every year from 1997 - 2008
  - Eligibility: 12 years ≤ age on December 31, 1997 ≤ 17 years
- Full data access: https://www.nlsinfo.org/investigator/

LEVELS IN NLSY97

2



1

LEVEL 1 VS. LEVEL 2

- Level 1 is the smallest unit of analysis
  - Level 1 datapoints are different in every row
- Level 2 variables are constant for all level 1 variables that are "nested" in it
  - Level 2 variables will be constant across ≥ 2 rows in your data spreadsheet

DATA STRUCTURE

Level 1	Level 2	Level 1	Level 2	Level 1
Respondent ID	School ID	Sex	School Type	Closeness With Best Friend
105	201	Male (1)	Private (1)	8
149	201	Female (-1)	Private (1)	7
2	101	Male (1)	Public (-1)	10
11	101	Female (-1)	Public (-1)	8
16	101	Male (1)	Public (-1)	10
21	202	Male (1)	Public (-1)	8
...	...	...	...	...

SOME FUNDAMENTALS

- Explaining variance in your outcome
- Dependence: Correlation and Covariance
- What if we ignored dependence in our data?

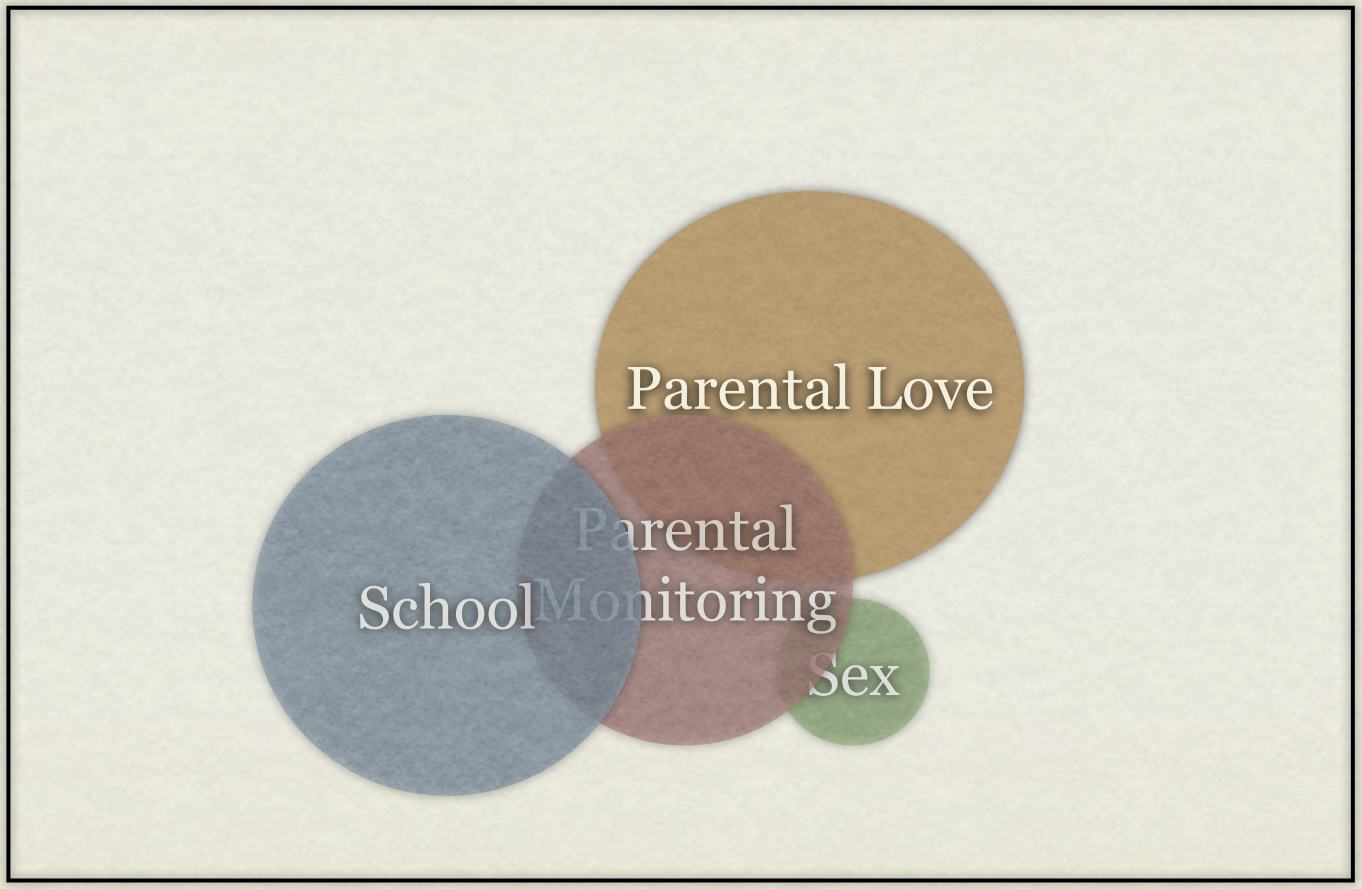
EXPLAINING VARIANCE IS THE GRAND PRIZE

- Almost any classical statistic compares:

$$\frac{\text{Variance Explained by your Independent Variable(s)}}{\text{Unexplained Variance}}$$

VARIANCE

Total Variance in Closeness with Best Friend



WHAT'S SO SPECIAL ABOUT VARIANCE?

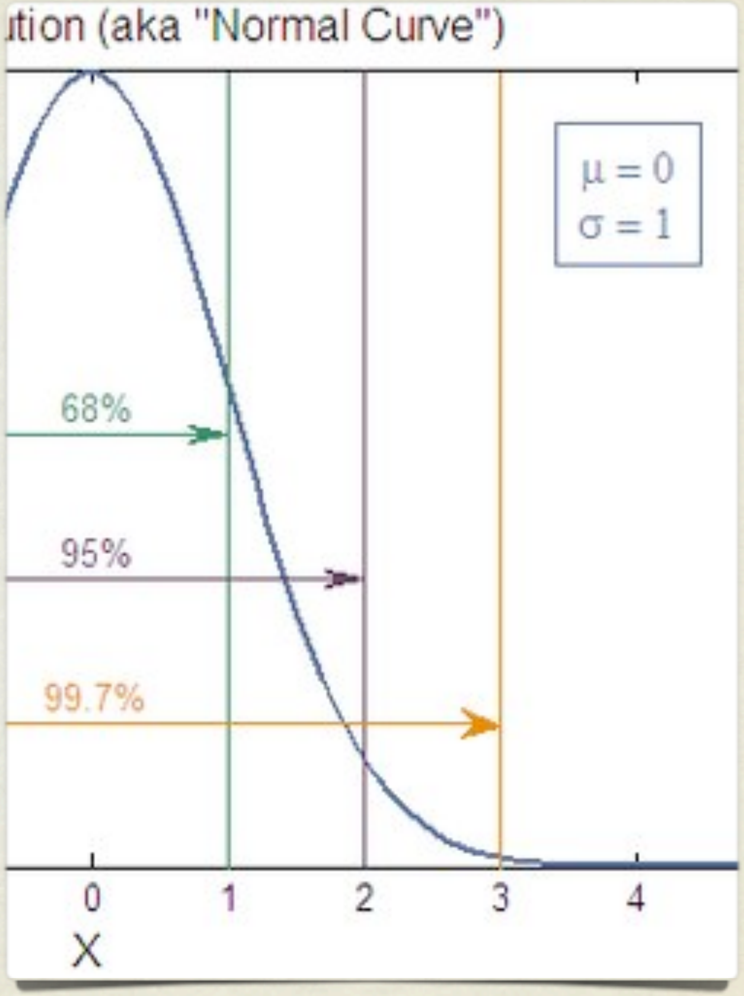
- Classical statistic's "Standard Candle"
  - Standard deviations are the unit of measurement
  - We know the *probability* of observing any degree of deviation from the mean

IMPORTANT ASSUMPTIONS OF CLASSICAL STATISTICS

- Assumptions:
  - Data were collected through **random sampling**
  - All data are **normally distributed**
  - **Variance must be equal** across conditions
  - All observations must be **independent**
- If your data violate an assumption:
  - *Transform it if you can, or*
  - *Accept a decrease in power if you can, or*
  - *Find a test that doesn't require it*

KEY ASSUMPTION RELATIVE TO MLM:

- **All observations must be independent**



DEPENDENCE IS WHERE IT'S AT

HOW DO WE MEASURE DEPENDENCE?

- Correlation & covariance

CORRELATION & COVARIANCE

- *How much 2 variables change together*

CORRELATION (r_{XY})

$$r_{XY} = \frac{\sum_{i=1}^{N} z_{x_i} z_{y_i}}{N-1}$$

- *Strength of predictive relationship between X and Y*
- Dimensionless

COVARIANCE (COV_{XY})

$$COV_{XY} = r_{XY}\sigma_X\sigma_Y$$

- *Unstandardized measure of relationship between X and Y*
- Values are in units of "XY"

WHY DO WE CARE?

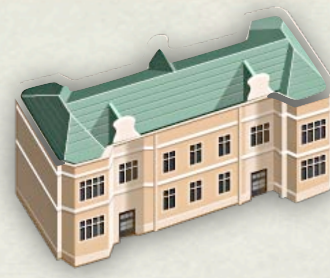
- Covariation *is* your dependence!

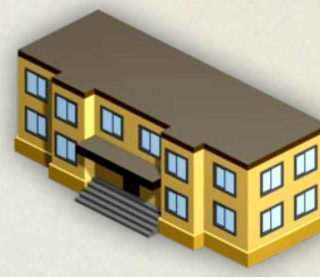
$$COV_{X_{Leafs_1} Y_{Leafs_2}} \qquad COV_{X_{Habs_1} X_{Habs_2}} \qquad COV_{X_{Pens_1} Y_{Pens_2}}$$

WHY DO WE CARE?

- Covariation _is_ your dependence!

2





$COV_{X_{101_1} X_{101_2}}$

$COV_{X_{201_1} X_{201_2}}$

$COV_{X_{202_1} X_{202_2}}$

1

OLD COPING METHODS

- *Groups suck; pretend they don't exist*
  - Use any GLM with no regard for group status
  - Use any GLM with group status as control variable
    - You are still violating assumptions of independence
- *Aggregate*

REGRESSION

- Estimation
- Moderated regression

REGRESSION

Predicted Value

Grand Mean of Y

Influence of X on Y

Stuff You Can't Explain

$$\boxed{\hat{y}_i} = \boxed{b_0} + \boxed{b_1} x_i + \boxed{e_i}$$

REGRESSION AND NLSY97

```
REGRESSION

/DEPENDENT life.satisfaction

/METHOD=ENTER
parental.love.
```

```
summary(  lm( life.satisfaction
~ parental.love ) )
```

Life Satisfaction=	b_0+b_1(Parental Love)
Estimate	= 3.97 + .135 (parental.love)

REGRESSION

	Grand Mean(Y)	Influence of X	Stuff You Can't Explain

$$y_i = \boxed{b_0} + \boxed{b_1} x_i + \boxed{e_i}$$

$$\hat{y}_i = \boxed{b_0} + \boxed{b_1} x_i$$

ESTIMATION

$$y_i = 3.97 + 0.135 x_{Ashif} + e_{Ashif}$$

i = Ashif

$$x_{Ashif} = -1$$

$$y_{Ashif} = 4$$

$$\hat{y}_{Ashif} = 4.11$$

$$\boxed{e_{Ashif} = .11}$$

MODERATED REGRESSION

- *Regression where the effect of one independent variable depends on another independent variable*

- Allows you to examine ***main effects*** and ***interactions***

$$\hat{y}_i = b_0 + \underbrace{\boxed{b_1 x_{1_i}} + \boxed{b_2 x_{2_i}}}_{\text{Main Effects}} + \underbrace{\boxed{b_3 (x_{1_i} * x_{2_i})}}_{\text{Interaction Effect}}$$

WHAT DOES IT *MEAN?*

- Multiple regression equation:
  - Every "+" represents an additive, main effect
    - The effects of each variable, *independent of its relationship with the other predictors*
  - Every multiplication represents a *dependence between predictors*

$$\widehat{y_i} = b_0 + b_1 x_{1_i} + b_2 x_{2_i} + b_3\left(x_{1_i} * x_{2_i}\right)$$

REGRESSION AND NLSY97 DATA

```
COMPUTE c.bf.close = bf.close - 8.743073.

COMPUTE loveXclose = parental.love*c.bf.close.

EXECUTE.

REGRESSION

/DEPENDENT life.satisfaction

/METHOD=ENTER parental.love c.bf.close
loveXclose.
```

```
summary( lm( life.satisfaction ~ love*c.bf.close)
```

Life Satisfaction =	b_0+b_1(Parents Love)$+b_2$(BF Closeness)$+b_3$(Parents Love* BF Closeness)
Estimate	= 3.97 + .12 (parental.love) + .03 (bf.close) + .05 (parental.love* bf.close)

BUT WE HAVE TO DO SOMETHING ABOUT THE LEVELS

- You want to explain all the variance you can
- Statistical assumptions matter
  - If you break them, your p-values are not the actual probability of observing the value of the statistic you observed

WHAT TO DO ABOUT GROUPS?

- We *shouldn't* ignore them
  - Ignoring = more unexplained variance
  - Ignoring = inaccurate comparison distributions

SOLUTIONS

- Alternatives to MLM:
  1. Aggregate your level 1 variables
  2. Random effects models
- Multilevel Modelling!

AGGREGATED DATA

1. Within each group, calculate the averages of each Level 1 variable

2. Run your analysis with the aggregate variable

    - Each group is your case

NON-AGGREGATED DATA

Respondent ID	School ID	Sex	School Type	Closeness With Best Friend
105	201	1	1	8
149	201	-1	1	7
2	101	1	-1	10
11	101	-1	-1	8
16	101	1	-1	10
21	202	1	-1	8
...	...	...	...	...

AGGREGATED DATA

School ID	Sex	School Type	Closeness With Best Friend
101	0.03	-1	8
201	0.04	1	7
202	0.004	-1	10
203	0.111	-1	8
301	-0.031	1	10
302	-0.014	-1	8
...	...	...	...

BENEFITS OF AGGREGATING

- All your cases are independent!
  - Use whatever analysis you want
- The aggregated variables will have:
  - Fewer outliers
  - Smaller variance

CONS OF AGGREGATING

- ¡POWER!
  - Your *N* is now the number of groups, not observations
- Changing the *unit* of analysis changes the *meaning*
- Your predictive resolution decreases

DEMO: REDUCED POWER

RANDOM EFFECTS MODELS

- First form of multilevel modelling
- Types of Random Effects Models:
  - Random intercept model/random effects ANOVA
  - Random slope models
- What's random about the intercepts and slopes?
  - They are *predicted*
  - So they have *error*

WHY AM I TELLING YOU THIS?

- When you run an MLM, you have to declare:
  - Your fixed effects
  - Your random effects

$$\widehat{y_{ij}} = Fixed + Random$$

RANDOM INTERCEPT MODELS

$$\widehat{y_{ij}} = \widehat{b_{0_j}} + b_1 x_{1_{ij}}$$

- = *random-effects ANOVA*
- A unique intercept is predicted for each group

RANDOM SLOPE MODELS

$$\widehat{y_{ij}} = b_0 + \widehat{b_{1_j}} x_{1_{ij}}$$

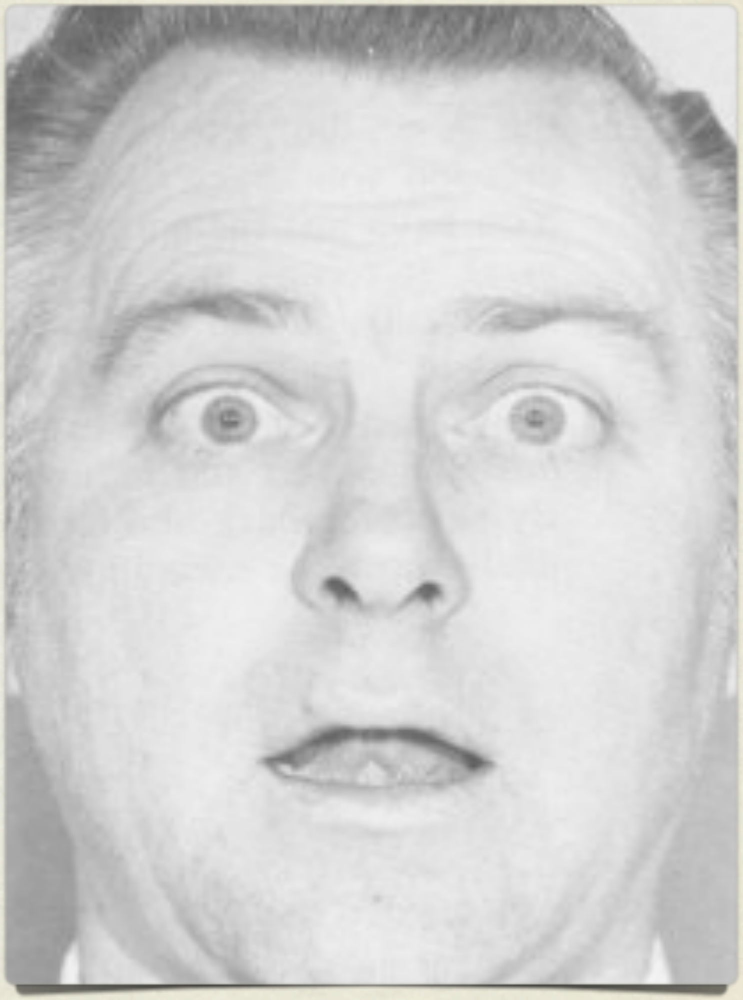
- A unique slope is predicted for each group

WHAT VARIES BETWEEN YOUR GROUPS?

- Their averages (= *random intercept*)
- Their change (= *random slope*)

WHOA!

- You just learned multilevel modelling!
- Random effects models *are* multilevel models



MULTILEVEL MODELS

MULTILEVEL MODELS!

- Putting it all together
- The equations
- Running a multilevel model

PUTTING IT ALL TOGETHER

- In regression you just estimate the outcome, $\widehat{y_i}$
- In MLM, you estimate parameters on the right side of the equation, too:
  - Intercept: $\widehat{b_0}$
  - Slopes: $\widehat{b_1}, \widehat{b_2}, \ldots$

REGRESSION & MLM

Regression: $$\widehat{y_i} = b_0 + b_1 x_i$$

MLM: $$\widehat{y_{ij}} = \hat{b}_{0_j} + \hat{b}_{1_j} x_{ij}$$

WHY DOES THIS SOLVE OUR PROBLEM?

- All unexplained variance: $\widehat{y_i} - y_i$
- We want to explain more of it by considering groups, $\widehat{y_{ij}} - y_{ij}$
  - Since each group j has its own intercept and/or slope, you are more accurate at predicting $\widehat{y_{ij}}$ for any individual in the group
  - Moreover, you are now accounting for the shared variance among group members

STANDARD NOTATION

- β = a predicted Level-1 parameter (i.e., intercept or slope)
- W = the group ID numbers
- γ = a Level-2 parameter (intercept or slope)
- Subscripts:
  - $\beta_{\_\,\_} = \beta_{\underline{\text{Index of Level 1 Parameter}}\ \underline{\text{Index of Level 2 Parameter}}}$
    - 0 is the index of the intercept
    - 1, 2, and 3 (and so forth) are the indexes of the first, second, and third slope parameters, respectively

MULTILEVEL MODEL

$$\widehat{y_{ij}} = \widehat{b_{0_j}} + \widehat{b_{1_j}} x_{1_{ij}} + e_{ij}$$

$$\beta_{0_j} = \gamma_{00} + \gamma_{01} W_j + u_{0_j}$$

$$\beta_{1_j} = \gamma_{10} + \gamma_{11} W_j + u_{1_j}$$

THE EQUATIONS

- Every predicted parameter has:
  - An equation that predicts it
  - Some error in this prediction
- Two ways to mathematically represent MLM:
  - “Multilevel Equations”
  - “Mixed Model”

"MULTILEVEL EQUATIONS" FORMAT

$$y_{ij} = \quad \textit{Fixed} \quad + \textit{Random}$$

$$y_{ij} = \boxed{\beta_{0_j} + \beta_{1_j} x_{1_{ij}}} + \boxed{e_{ij}}$$

$$\beta_{0_j} = \boxed{\gamma_{00} + \gamma_{01} W_j} + \boxed{u_{0_j}}$$

$$\beta_{1_j} = \boxed{\gamma_{10} + \gamma_{11} W_j} + \boxed{u_{1_j}}$$

"MIXED MODEL" FORMAT

$$y_{ij} = \qquad Fixed \qquad + \quad Random$$

$$y_{ij} = \boxed{\left(\gamma_{00} + \gamma_{01}W_j + \gamma_{10}x_{1_{ij}} + \gamma_{11}W_j x_{1_{ij}}\right)} + \boxed{\left(e_{ij} + u_{0_j} + u_{1_j}x_{1_{ij}}\right)}$$

HOW DO I ESTIMATE THE PARAMETERS?

- Thankfully, a computer does it for you using an iterative process that minimizes residuals for all estimated parameters
- This process relies on the covariance matrix of individuals within groups
- This process also determines your degrees of freedom

COVARIANCE MATRICES

- Covariance matrix
  - *Assumed relationship among Level 1 data points from the same Level 2 group*
- Most widely used covariance matrices:
  - Variance Components - Default in SPSS and SAS, assumes that data points from different groups do not covary
  - Autoregressive - Standard for basic longitudinal designs, assumes that data points next to each other will be highly correlated
  - Unstructured - Only option in R (thus the default), assumes nothing about covariation structure, best for complicated multilevel models, robust against issues like heteroskedasticity
- Great online resource: http://courses.ttu.edu/isqs5349-westfall/images/5349/mixed_covariance_structures.htm

COVARIANCE MATRICES

- The covariance matrix of a multilevel defines:
  - *How observations from the same group relate to one another*
- Easy defaults:
  - Only modelling a random intercept:
    - Use "*Variance Components*"
  - Repeated-measures data (e.g., diaries):
    - Use *"Autoregressive"* covariance matrix
  - Any complex structure (e.g., both between- and within- random effects):
    - Use "*Unstructured*" covariance matrix

ESTIMATING DEGREES OF FREEDOM

- The degrees of freedom (df) are *estimated* in MLM based on the iteration process
- Most common df estimation methods in MLM:
  - <u>Satterthwaite</u> - Default in SPSS and SAS and most widely-used method
    - Akin to a classic ANOVA or regression
    - Note that your *df* will have **decimal** points
  - <u>Between-Within</u> - Only method used by R (and thus the default), more conservative, is robust to complex hierarchical structures

RUNNING A MULTILEVEL MODEL

FUNDAMENTAL PRELIMINARIES

- All predictors should be mean-centered (continuous) or effect-coded (categorical)
  - Continuous: X = X - mean(X)
  - Effect-coding: if X = "Stimulus Type A", then X = -1

    if X = "Stimulus Type B", then X = 1

MLM SYNTAX!

- **Random Intercept only:**

```
MIXED life.satisfaction WITH parental.love
 /FIXED= parental.love
/PRINT= SOLUTION
/RANDOM=INTERCEPT | SUBJECT( school.id ).
```

```
lme( life.satisfaction ~ parental.love, random=~1 | school.id )
```

MLM SYNTAX!

- **Random Intercept & Random Slope:**

```
MIXED life.satisfaction WITH parental.love

 /FIXED= parental.love

/PRINT= SOLUTION

/RANDOM=INTERCEPT parental.love | SUBJECT( school.id ).
```

```
lme( life.satisfaction ~ parental.love, random=~1+parental.love |
school.id )
```

LET'S TEST OUR MODERATION QUESTION PROPERLY …

- When we assume that students who go to the same school are more similar to each other …
  - … are the effects of parents love on life satisfaction dependent on closeness with one's best friend?

GROUPING VARIABLES

- How many levels?
- What is nested in what?

GROUPING IN NLSY97

- Level 1: Each *Student's* parental love, closeness with best friend, and life satisfaction in late 20s

EFFECTS SPECIFICATION

- Fixed versus random effects
- Covariance matrices
- Method for estimating degrees of freedom

FIXED V. RANDOM EFFECTS

- What are your model's random effects?
  - Are you modelling random intercepts only?
  - Are you modelling random intercepts *and* slopes?

FIXED V. RANDOM EFFECTS IN NLSY97

- Fixed:
  - Best friend closeness
  - Parental love
  - School Type (Public/Private: Level 2 Covariate)
- Random:
  - Intercept for each school

COVARIANCE MATRIX FOR NLSY97

- Decision: *Variance Components*
- Reason:
  - Only estimating a random intercept for each school
  - Assumes that life satisfaction of Americans who attended the same high school are correlated with each other

DEGREES OF FREEDOM ESTIMATION

- The method of *df* estimation in a multilevel model determines how *df* are estimated
- Easy default: Satterthwaite
  - Most similar to a classical analysis
- If you want to be conservative: Between-within
  - Will give you the lowest degrees of freedom for tests of parameter estimates

DF ESTIMATION IN NLSY97

- Decision: *Satterthwaite*
- Reason:
  - Simple data structure: Only estimating a random intercept for each school
  - We are just using MLM to avoid violating statistical assumptions, and thus want an analysis most similar to classic regression

RUN THE MODEL!

- **Syntax:**

```
MIXED life.satisfaction WITH bf.close parental.love

/FIXED=bf.close parental.love bf.close*parental.love

/RANDOM=INTERCEPT | SUBJECT(school.id)

/PRINT=SOLUTION.
```

```
lme( life.satisfaction~bf.close*parental.love, random=~1 |
school.id )
```

OUTPUT

- Look for:
  - Fixed effects table
  - Random effects table
  - Model evaluation criteria

REPORTING YOUR ANALYSIS

- What people want to know:
    - The type of multilevel model you conducted (e.g., random intercept? Random slope?)
    - Your “nesting” variable (Level 2 Grouping Variable)
    - Your DV, IVs, and covariates
    - What covariance matrix you used
    - The method of estimating degrees of freedom

REPORTING NLSY97 ANALYSIS

- Model specification: DV, IVs, and covariates
  - *"Life satisfaction was modeled as a function of students' closeness with best friend, feelings of love from parent, and their interaction."*
- Type of multilevel model conducted
  - *"A 2-level multilevel model was used …"*
- Nesting variable with random effects stated
  - *"… to account for students nested within school by estimating a random intercept for each school …"*
- What covariance matrix and *df* estimation method you used
  - *"… using the variance components covariance structure and the Satterthwaite method of estimating degrees of freedom."*

REPORTING YOUR RESULTS

- Statistic

  1. The fixed effects for any parameter that you estimated (e.g., *b*) and its associated standard error, *SE*

  2. The statistic that tests whether the parameter is different from 0 (e.g., *t*, *F*) and the associated degrees of freedom

  3. Probability of observing that statistic

- (If Moderation:) Results of simple effects testing (see West, Aiken, & Krull, 1996)

- Visualization

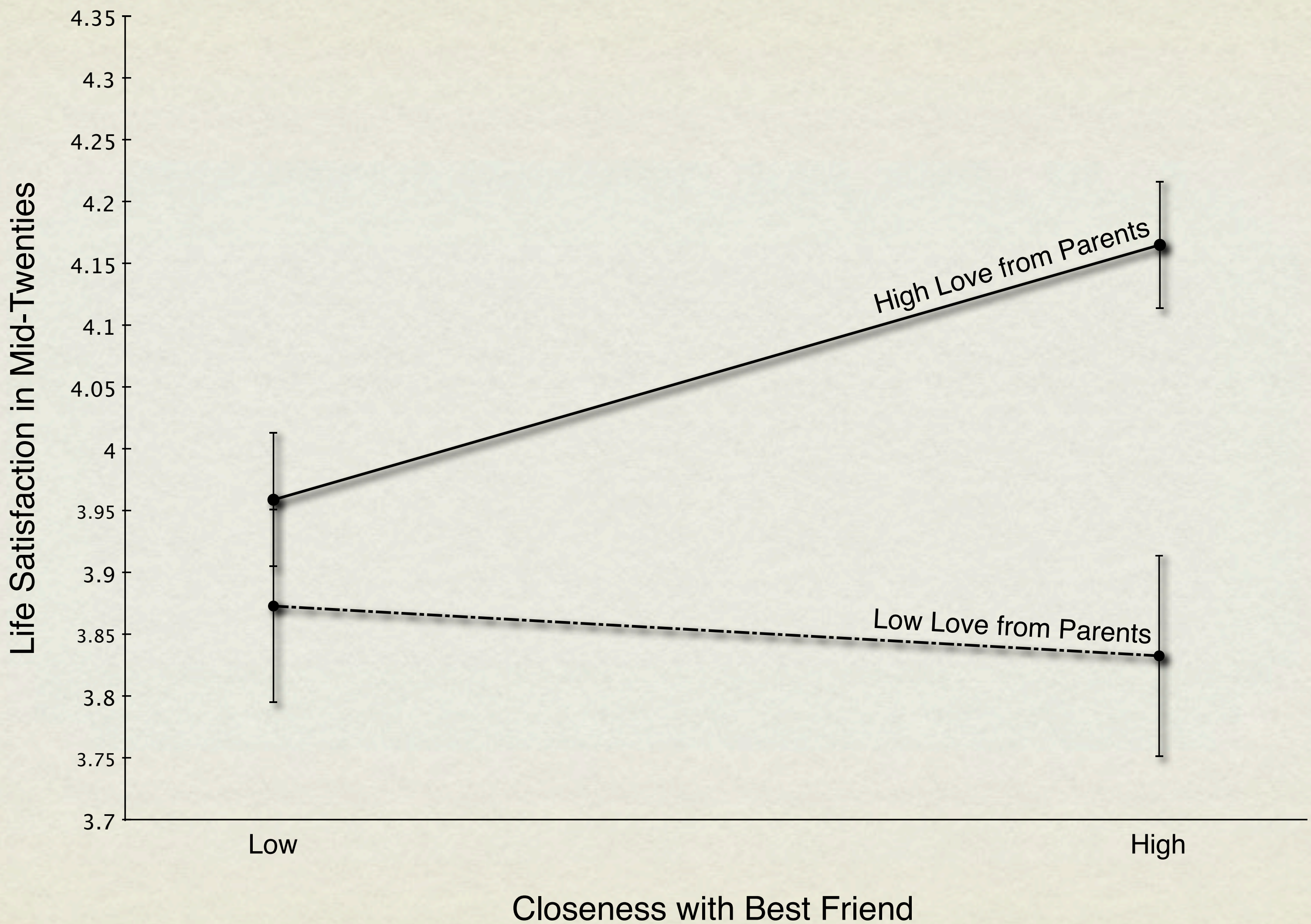
REPORTING RESULTS FOR NLSY97

- *Main effects - Report F- or t-values of Fixed Effects:*
  - "There was no main effect of closeness with best friend, b = 0.03, SE = 0.02, t(806) = 1.19, p = .23."
  - "There was a significant main effect of parental love on life satisfaction, b = 0.13, SE = 0.04, t(806) = 3.47, p < .001."

REPORTING RESULTS FOR NLSY97

- *Interaction - Report t- or F-value of fixed effect*
    - "As shown in Figure 1, best friend closeness significantly moderated the relationship between parental.love and life satisfaction, b = -0.000046, SE = .000012, $t(474) = -3.82$, $p < .001$."
- Simple Slopes - Report t-values of fixed effects
    - "Simple slopes were examined at one standard deviation above and below the means of both predictors (Aiken & West, 1991). This analysis revealed that participants with low parental love have no boost in life satisfaction as a function of the closeness with their best friend, $t(806) = -0.38$, $p = 0.70$, but best friend closeness predicted greater life satisfaction among participants with high parental love, $t(806) = 2.86$, $p = .004$. Parental love did not predict life satisfaction among participants who were low in closeness with their friends, $t(806) = 1.02$, $p = .31$, but parental love was strongly associated with life satisfaction among participants who felt close to their best friend, $t(806) = 3.85$, $p = .0001$."

LIFE SATISFACTION AS A FUNCTION OF FAMILY AND PEER RELATIONSHIPS



BENEFITS OF MLM

- ***Theoretical***: More accurately captures reality
- ***Statistical***:
  - Statistical integrity
  - Greater power than aggregating
  - More variance explained!
- ***Pragmatic***: Editors may require it
- ***Tertiary***: It sounds cool

MORE VARIANCE EXPLAINED!

= significance

= publications

= job security

EFFECT SIZE AND POWER IN MLM

EFFECT SIZE IN MLM

- Unstandardized coefficients & standard errors
- Variance explained

UNSTANDARDIZED COEFFICIENTS & SE

- *The significance of each fixed effect*
  - Unstandardized b and its SE tells you how reliable your effect is
  - Your t value is simply a ratio of b/SE

VARIANCE EXPLAINED

- R^2 has slightly different meaning between regression and MLM
- R^2 in normal regression
  - *Percentage of the dependent variable's variance that is explained by the predictor variables*
- R^2 in multilevel modelling
  - *Proportional reduction in prediction error*

Kreft & de Leeuw (1998); Snijders & Bosker (1994, 1999)

EVALUATING R^2 IN MLM

- Calculate an R^2 at each level
- Interpretation:
  - Level 1 R^2
    - *Reduction of prediction error provided by your independent variables when predicting the outcome*
  - Level 2 R^2
    - *Reduction in prediction error provided by your independent variables when explaining why groups differ from one another*
  - Use same criteria as normal R^2 to identify small, medium, and large R^2
    - Cohen (1992): Small $R^2 = 0.02$, Medium $R^2 = 0.13$, Large $R^2 = 0.26$

Kreft & de Leeuw (1998); Snijders & Bosker (1994, 1999)

R^2 IN MLM

1. Run your multilevel model
   - Note the residual & intercept variances
2. Run the "baseline model"
   - *Baseline model is a multilevel model with no predictors and only a random intercept*
   - Note the residual & intercept variances
3. Insert these values into an equation

Kreft & de Leeuw (1998); Snijders & Bosker (1994, 1999)

HOW TO RUN THE BASELINE MODEL

- **SPSS**

```
MIXED life.satisfaction
/FIXED=INTERCEPT
/RANDOM=INTERCEPT | SUBJECT(school.id)
/PRINT=SOLUTION.
```

- **R**

```
lme( life.satisfaction ~1, random=~1|school.id )
```

R^2 IN MLM

- Two methods, both reasonably equivalent:
    - Snijders & Bosker (1994, 1999) approach
        - Most robust and widely used

        ➡*Use this method first*
    - Kreft & de Leeuw (1998) approach
        - Is less robust against multivariate non-normality
        - But is more flexible and easier to calculate
- If either gives you a negative value, try the other method. If they both give you negative values, set the effect size equal to 0

R^2 USING SNIJDERS & BOSKER METHOD

Estimate of Level 1

Variance

$$R_1^2 = 1 - \frac{\left(\sigma_{u_0}^2 + \sigma_r^2\right)_{Comparison}}{\left(\sigma_{u_0}^2 + \sigma_r^2\right)_{Baseline}}$$

Estimate of Level 2

Variance

$n_{\text{observations}}$

within-group

$$R_2^2 = 1 - \frac{\left(\sigma_{u_0}^2 + (\sigma_r^2 / n)\right)_{Comparison}}{\left(\sigma_{u_0}^2 + (\sigma_r^2 / n)\right)_{Baseline}}$$

Snijders & Bosker (1994, 1999)

R^2 FOR NLSY97 DATA

$$R_1^2 = 1 - \frac{(.000796 + 0.745)}{(1.23\text{e-}08 + 0.767)}$$

$$R_1^2 = 1 - \frac{.746}{.767}$$

$$R_1^2 = 1 - .973 = .03$$

R^2 FOR NLSY97 DATA

$$R_2^2 = 1 - \frac{\left(.000796 + (0.745/176)\right)}{\left(1.23\text{e-}08 + (0.767/176)\right)}$$

$$R_2^2 = 1 - \frac{(.000796 + .00423)}{(1.23\text{e-}08 + .00436)} = 1 - \frac{.005}{.00436}$$

$$R_2^2 = 1 - 1.15 = \cancel{-.15} = 0$$

R^2 USING KREFT & DE LEEUW METHOD

$$R_1^2 = \frac{\left(\sigma_r^2\right)_{Baseline} - \left(\sigma_r^2\right)_{Comparison}}{\left(\sigma_r^2\right)_{Baseline}}$$

$$R_2^2 = \frac{\left(\sigma_{u_0}^2\right)_{Baseline} - \left(\sigma_{u_0}^2\right)_{Comparison}}{\left(\sigma_{u_0}^2\right)_{Baseline}}$$

Kreft & de Leeuw (1998)

R^2 FOR NLSY97 DATA

$$R_1^2 = \frac{0.767 - 0.745}{0.767}$$

$$R_1^2 = \frac{.022}{.767}$$

$$R_1^2 = 0.03$$

Kreft & de Leeuw (1998)

R^2 FOR NLSY97 DATA

$$R_2^2 = \frac{1.23\text{e-}08 - .000796}{1.23\text{e-}08}$$

$$R_2^2 = \frac{-.000796}{.0000000123}$$

$$R_2^2 = \cancel{-64715} = 0$$

Kreft & de Leeuw (1998)

REPORTING R^2

- We calculated the proportion reduction in error of the model for each level according to the recommendations of Snijders & Bosker (1994, 1999). At the level of the respondent, the model reduced prediction error of pro-tobacco voting by a small amount for any given student, $R^2{}_1 = .03$. At the level of the school, the model did not reduce the prediction error of the baseline model for each school, $R^2{}_2 = 0$.

POWER IN MLM

- Once you know your effect size, power is easy to calculate
  - Cohen (1992) - available in “Supplemental Readings”
- But the relevant sample size depends on whether your predictors are above Level 1 or not

POWER IN ALL-LEVEL 1 MLM MODELS

- If all your predictors are at level 1 (e.g., life.satisfaction ~ parental.love + bf.close)
  - ~30 observations total for a large effect
  - ~70 observations total for a medium effect
  - ~85 observations total for a small effect

POWER IN MLM MODELS WITH LEVEL 2 PREDICTORS

- If some or all of your predictors are at Level 2 (e.g., life.satisfaction ~ school.type) ...
  - ~30 groups for a large effect
  - ~70 groups for a medium effect
  - ~85 groups for a small effect

DO I EVEN NEED TO USE MLM?

THE INTRACLASS CORRELATION COEFFICIENT

INTRACLASS CORRELATION (ICC)

- *A measure of how dependent observations within a group are on each other*
- You calculate the ICC **from the baseline model**

Estimate of Level 2 Variance

$$\rho = \frac{\sigma_{u_0}^2}{\sigma_{u_0}^2 + \sigma_r^2}$$

Estimate of Level 1 Variance

INTERPRETING ICC

- If ICC is significant …
  - *Residuals are correlated more within groups than between groups*
  - You must use MLM or otherwise take this covariation into account
- If ICC is not significant …
  - You have the <u>*option*</u> not to use MLM; you can use any test that assumes independence between observations
  - You can still use MLM with legitimacy, though

ICC FOR NLSY97 DATA

$$\rho = \frac{0.000111}{0.000111 + 0.876}$$

$$\rho = .0001$$

- Compare the value of ρ to published significance tables for the correlation coefficient, r, using your Level 1 n to determine significance (*hint*: Google for "calculate significance correlation")
- **Conclusion:**
  - *"The intraclass correlation coefficient was not significant, ρ = 0.0001, $t(816) = 6.74$, $p < .001$, suggesting that the life satisfaction ratings of students from the same school were independent of each other. Nonetheless, we conducted all analyses using multilevel modeling to acknowledge the natural hierarchy in the data."*

THINGS TO CONSIDER

THINGS TO ALWAYS KEEP IN MIND

- Research design
- Normality of data
- Unstandardized coefficients

RESEARCH DESIGN

- Things to always remember:
  - Measure the same variables for every observation
  - Make sure to record the grouping variable
    - An ID number for each group
  - Think about your model BEFORE you collect your data
    - Try to make the levels as clear cut as possible

NORMALITY

- Normality is important in regular regression, and paramount in multilevel modelling
  - Non-normality usually hurts your power
- If your data are not normal:
  - Skewed: Transform them
  - Heteroskedastic: Use *unstructured* covariance matrix
  - Non-Gaussian: Use a generalized linear model (more on this next week)

UNSTANDARDIZED COEFFICIENTS

¡¡DO <u>NOT</u> STANDARDIZE ALL YOUR VARIABLES BEFORE RUNNING A MULTILEVEL MODEL!!

USE UNSTANDARDIZED VARIABLES

- It totally messes the whole thing up
  - Your model falsely converges on a solution right away
  - Your slopes and intercepts are *wrong*
- No matter how badly you want standardized coefficients, just don’t do it

ADVANCED APPLICATIONS OF MLM

MLM APPLICATIONS

- *N*-level models
- Nested Growth Curves
- Cross-classification
- Multilevel mediation

N-LEVEL MODELS

- Theoretically, you can run models with any number of levels
  - 2-levels - e.g., children nested in classrooms
  - 3-levels - e.g., children nested in classrooms that are nested in schools
  - 4-levels - e.g., children nested in classrooms that are nested in schools that are nested in school boards
  - 5-levels - e.g., children nested in classrooms that are nested in schools that are nested in school boards that are nested in provinces
  - 6-levels - e.g., children nested in classrooms that are nested in schools that are nested in school boards that are nested in provinces/states that are nested in countries
  - ... the limit is decided by your data and your design

N-LEVEL MODELS

- Run a multilevel model with more than 2 levels when:
  - *You have grouping variables that are also clustered together*
- How to implement:
  - Model one random intercept (and/or slope) for the highest-level group
  - Model another random intercept (and/or slope) for each combination of lower-level groups in the higher-level groups

MORE LEVELS IN NLSY97

3

2

1

3-LEVEL MODELS IN NLSY97

- Highest Level (3):
    - School ID
- Middle Level (2):
    - Participant ID
- Lowest Level (1), 4 measurements per participant:
    - Time ID

HOW TO RUN A 3-LEVEL MODEL

- **SPSS**

```
MIXED grades WITH love.mlm discipline.mlm

/FIXED= love.mlm discipline.mlm love.mlm*discipline.mlm

/RANDOM=INTERCEPT  | SUBJECT(school.id*id)

/RANDOM=INTERCEPT  | SUBJECT(school.id)

/PRINT SOLUTION.
```

- **R**

```
lme( grades ~ love.mlm*discipline.mlm, random=~1|school.id/id )
```

A WORD OF CAUTION

- But, if you want to use predictors from the higher levels ...
  - ... you must have sufficient number of groups at that level to find significance with your effect size
  - e.g.,
    - ~30 groups for a large effect
    - ~70 groups for a medium effect
    - ~85 groups for a large effect

GROWTH CURVES

- You have:
  - Multiple observations across time
  - You expect that people *change at different rates*
- Is it a *nested* growth curve or just a growth curve?
  - If your participants are also nested in a grouping variable, it is a nested growth curve

NESTED GROWTH CURVES

- How to implement:
  - Record measurement number (e.g., “time”)
  - Include “time” as a predictor in your fixed effects model
  - Include the slope of “time” as a random effect
  - Use an unstructured covariance matrix and between-within degrees of freedom

HOW TO RUN A NESTED GROWTH CURVE

- **SPSS**

  MIXED arrests WITH sex time consequences bf.close

  /FIXED= sex time consequences bf.close time*consequences time* bf.close consequences*bf.close time*consequences*bf.close

  /PRINT=SOLUTION

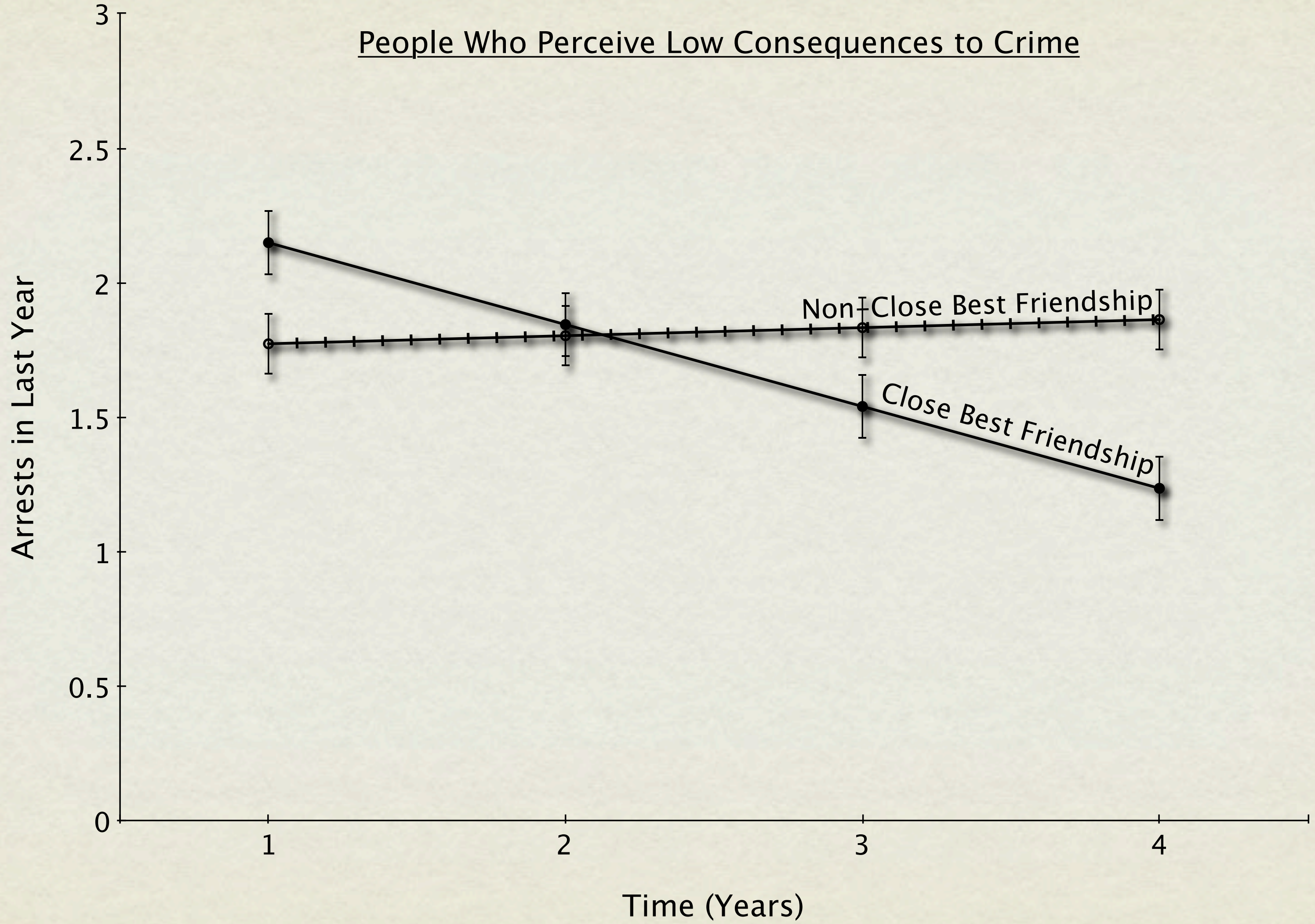
  /RANDOM=INTERCEPT time | SUBJECT(school.id) COVTYPE(UNR)

  /RANDOM=INTERCEPT time | SUBJECT(school.id*id) COVTYPE(UNR).

- **R**

  lme(arrests ~ sex.mlm + time*consequences*bf.close, random=~1+time|school.id/id)

FRIENDSHIP AS A BUFFER AGAINST CRIMINALITY WHEN THE PERCEIVED CONSEQUENCES OF CRIME ARE LOW



CROSS-CLASSIFICATION

- *A multilevel model with two grouping variables that are not nested in one another*
- Example:
  - Repeated measures are nested in both *participants* but also in the *interviewers* who collected the data
    - Participants typically had a different interviewer each year
  - The hierarchical relationship between participants and interviewers is unclear; they appear to be at the same level

CROSS-CLASSIFICATION

- *When you have more than 1 way you can nest your variables*
- How to implement:
  - Model a random intercept (or slope) for each group

HOW TO RUN A CROSS-CLASSIFIED ANALYSIS

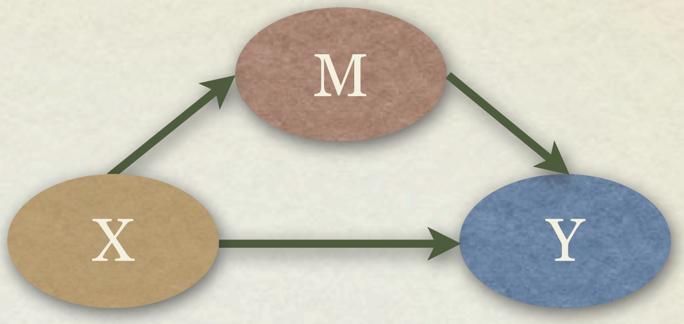
- **SPSS**

```
MIXED consequences WITH discipline

/FIXED= discipline

/RANDOM=INTERCEPT | SUBJECT(id)

/RANDOM=INTERCEPT | SUBJECT(interviewer.id)

/PRINT=SOLUTION.
```

- **R**

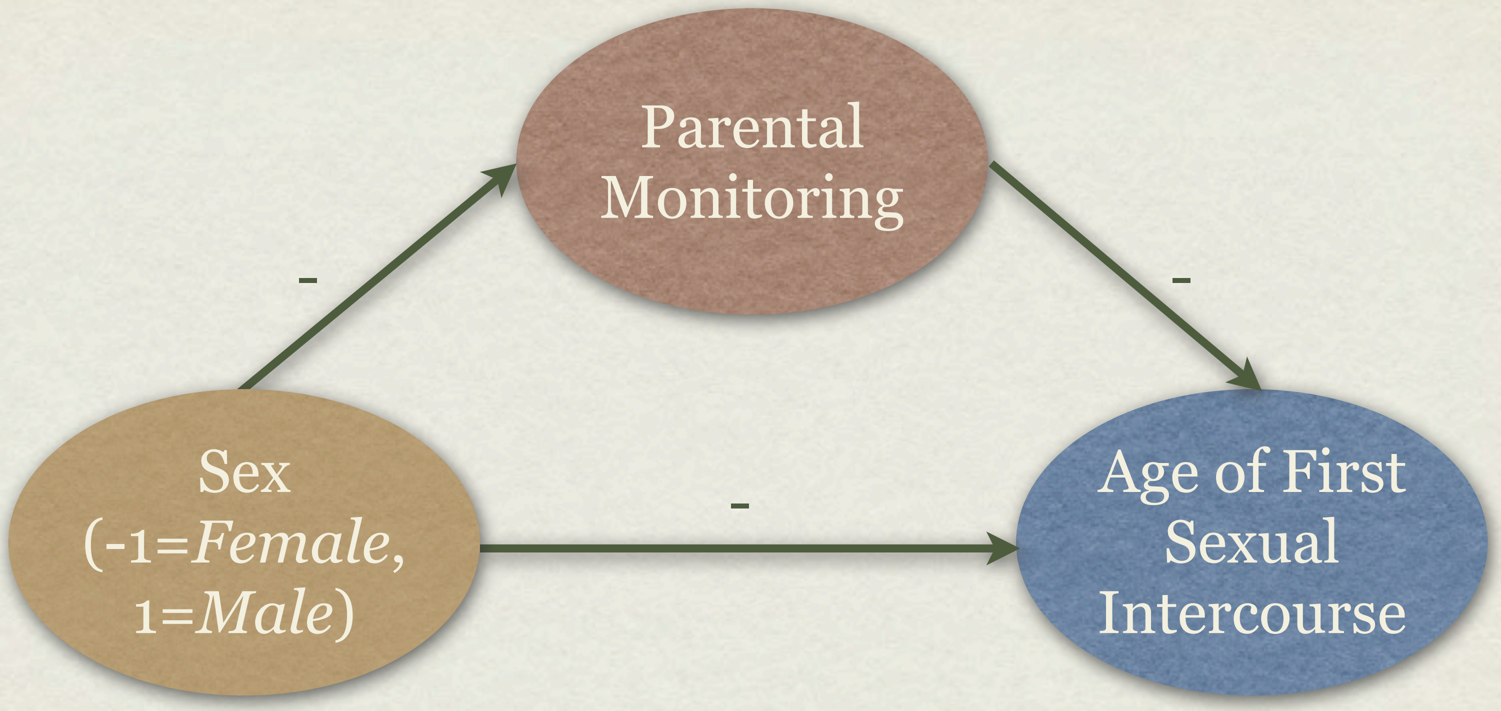
```
lmer( consequences ~ (1|id) + (1|interviewer.id) + discipline )
```

MULTILEVEL MEDIATION



- Similarities
  - Conducting the analysis
- Differences
  - Considering the levels
  - Determining applicability across groups

EXAMPLE MEDIATION IN NLSY97 DATA



RUNNING THE ANALYSIS

- Essentially, use the causal steps approach:
  - Sexual Age = Participant Sex
  - Parental Monitoring = Participant Sex
  - Sexual Age = Participant Sex + Parental Monitoring
- *BUT …*
  - … depending on the levels of your predictor and mediator, you may need some special covariates and a different approach to centering

Zhang, Zyphur, & Preacher (2009)

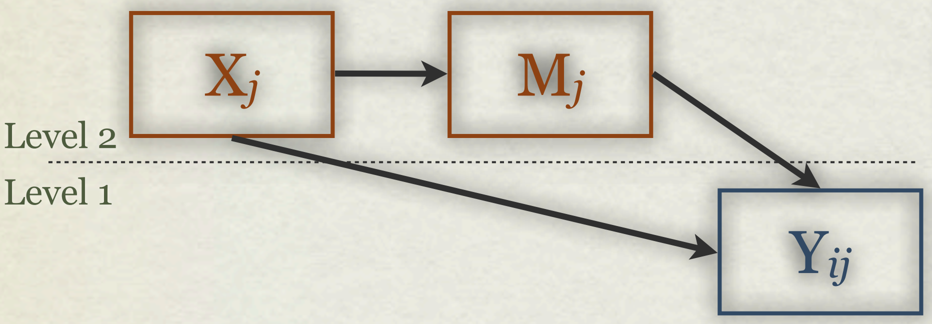
CONSIDERING THE LEVELS

- Ask yourself:
  - Is my predictor a Level 1 or Level 2 variable?
  - Is my mediator a Level 1 or Level 2 variable?
- You can use the normal Causal Steps (Baron & Kenny, 1986) approach when:
  - Both the Predictor and Mediator are Level 2 variables
- Otherwise, you need special covariates and a different approach to centering

Zhang, Zyphur, & Preacher (2009)

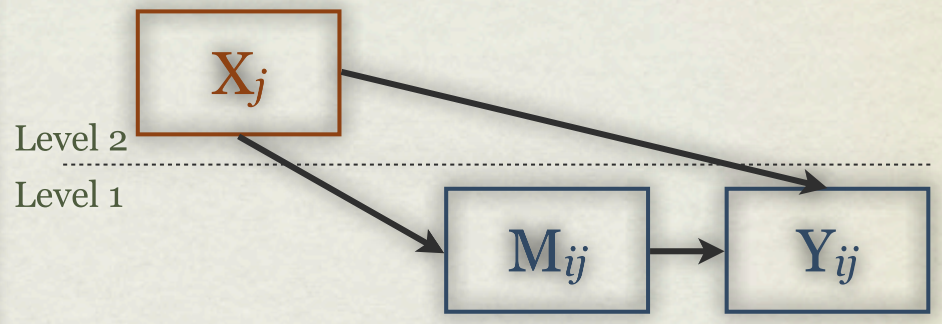
TYPES OF MLM MEDIATION

Normal Causal Steps Approach

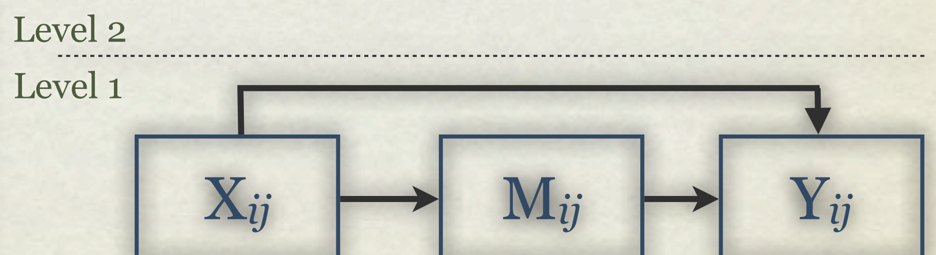


"2-2-1 Mediation"

Causal Steps With Extra Covariates

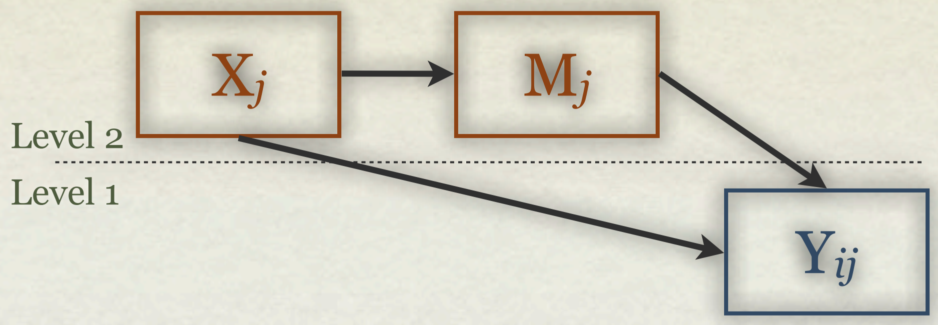


"2-1-1 Mediation"



"1-1-1 Mediation"

2-2-1 MEDIATION



- Your mediation effect is unconfounded with effects at the different levels … yay!

- Normal Causal Steps Approach:

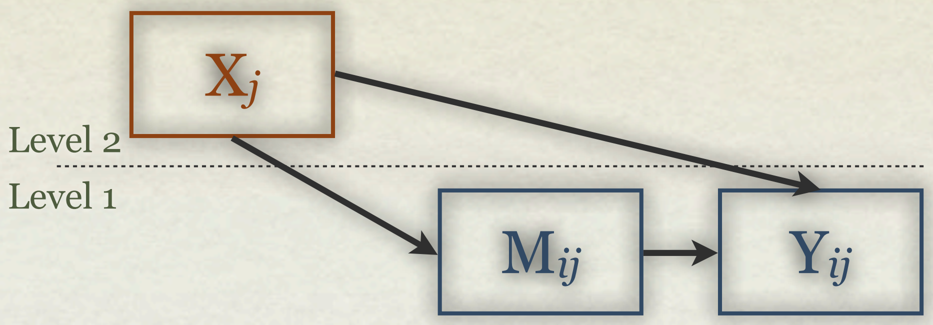
1. $M_j = (X_j$

2. $Y_{ij} = X_j$

3. $Y_{ij} = X_j + M_j$

Zhang, Zyphur, & Preacher (2009)

2-1-1 MEDIATION



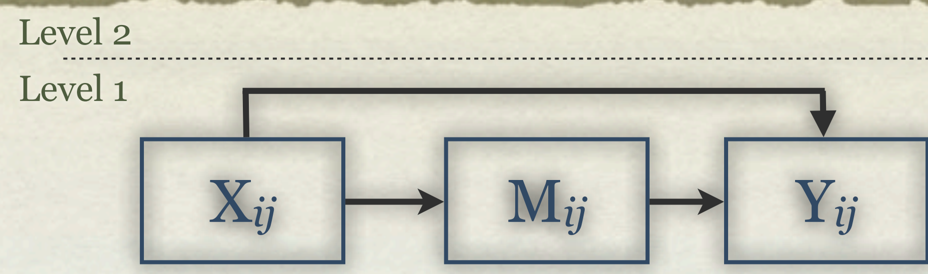
- Confounding across levels! Oh my!
    - It is possible that the relationship between the mediator and outcome is the result of either deviations of the *group* at Level 2 or the *individual from their group* at Level 1
- Amended Causal Steps approach with the group average for the mediator as a covariate & group-mean-centered Mediator:

1. $M_{ij} = X_j$
2. $Y_{ij} = X_j$
3. $Y_{ij} = X_j + M_j + (M_{ij} - M_j)$

- Look for effects at Level 2: Significance of X_j in Steps 1 and 3, and significance of M_j in Step 3

Zhang, Zyphur, & Preacher (2009)

1-1-1 MEDIATION



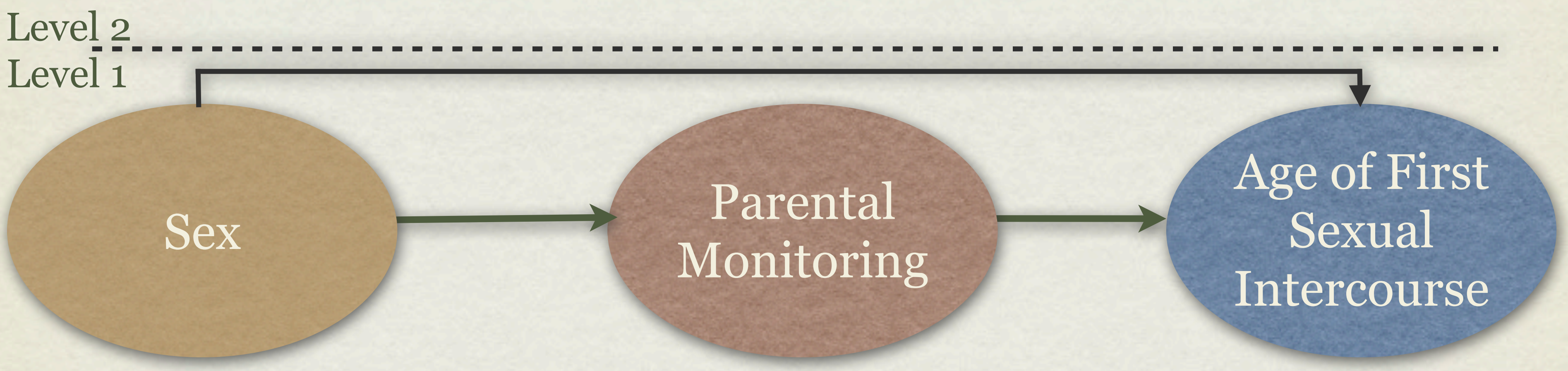
- Level 1 is confounded with Level 2! Oh my!
- Amended Causal Steps approach with the group averages for X and M & group-mean-centered values for the predictors, X and M:

  1. $M_{ij} = X_j + (X_{ij} - X_j)$
  2. $Y_{ij} = X_j + (X_{ij} - X_j)$
  3. $Y_{ij} = X_j + (X_{ij} - X_j) + M_j + (M_{ij} - M_j)$

- Look of significance of effects at *either* Level:
  - **Mediation at Level 1**: $(X_{ij} - X_j)$ in Steps 1 & 3 and $(M_{ij} - M_j)$ in Step 3, **OR**
  - **Mediation at Level 2**: X_j in Steps 1 & 3 and M_j in Step 3

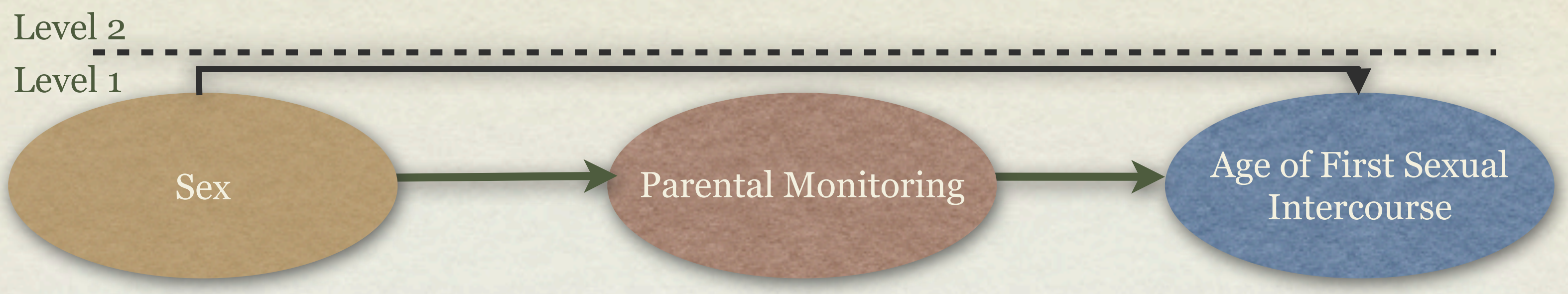
Zhang, Zyphur, & Preacher (2009)

MULTILEVEL MEDIATION IN NLSY97 DATA



“1-1-1 Mediation”

1-1-1 MEDIATION IN NLSY97 DATA



- First Steps

1. Create aggregate variables for sex and parental monitoring (e.g., "sex.agg", "parental.monitoring.agg")
2. Create group-mean centered ("CWC") variables for sex and parental monitoring using the aggregate variable you just created
3. Grand-mean center the aggregate variables prior to analysis

- Then run your models with those variables!

3. sexual.age = sex.agg + sex.cwc

4. parental.monitoring = sex.agg + sex.cwc

5. sexual.age = sex.agg + sex.cwc + parental.monitoring.agg + parental.monitoring.cwc

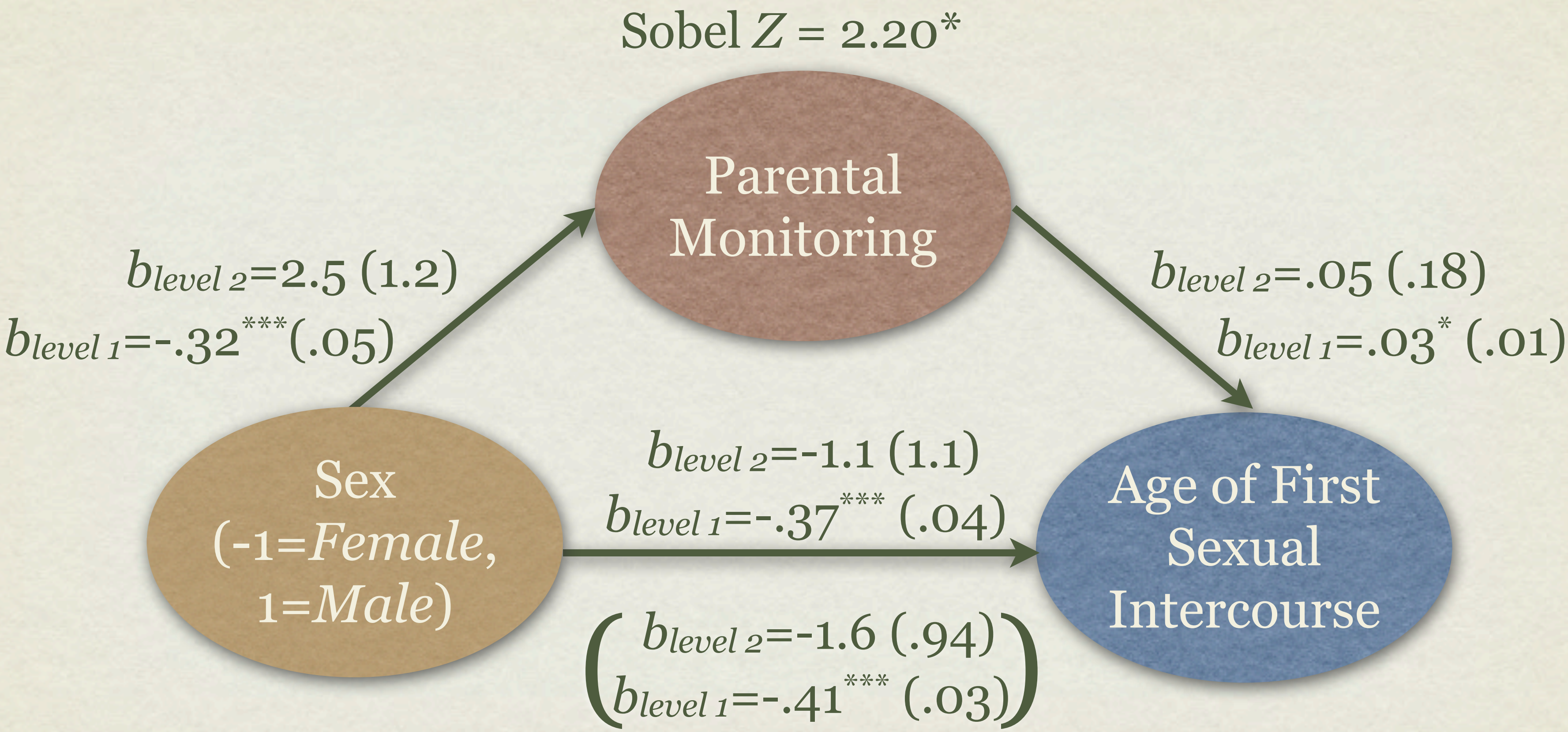
1-1-1 MEDIATION IN NLSY97 DATA

1. Create group average of sex and parental monitoring for each school
   - SPSS:
     - ```
       AGGREGATE
       /OUTFILE=* MODE=ADDVARIABLES
       /BREAK=school.id
       /sex.agg=MEAN(sex)
       /parental.monitoring.agg=MEAN(parental.monitoring).
       ```
   - R:
     - `AGGREGATE <- data.frame( school.id = levels( school.id ) )`
     - `AGGREGATE$parental.monitoring.agg <- tapply(parental.monitoring, school.id, mean, na.rm=T)`
     - `merged.data <- merge( nls.data, AGGREGATE, by="school.id", all.x=TRUE)`
     - `parental.monitoring.agg <- merged.data$parental.monitoring.agg`
       ```

1-1-1 MEDIATION IN NLSY97 DATA

2. For each participant, center their sex and parental monitoring around their school's mean

- SPSS:
  - `COMPUTE sex.cwc = sex - sex.agg.`
  - `COMPUTE parental.monitoring.cwc = parental.monitoring - parental.monitoring.agg.`
  - `EXECUTE.`
- R:
  - `sex.cwc <- sex - sex.agg`

    `parental.monitoring.cwc <- parental.monitoring - parental.monitoring.agg`

1-1-1 MEDIATION IN NLSY97 DATA

3. Center the aggregate variables around their grand mean

- SPSS:
  - `DESCRIPTIVES`

    `/VAR=sex.agg parental.monitoring.agg.`
  - `COMPUTE sex.agg.c = sex.agg - .0238.`

    `COMPUTE parental.monitoring.agg.c = parental.monitoring.agg - 8.7320.`

    `EXECUTE.`
- R:
  - `sex.agg.c <- sex.agg - mean(sex.agg, na.rm=T)`
  - `parental.monitoring.agg.c <- parental.monitoring.agg - mean(parental.monitoring.agg, na.rm=T)`

1-1-1 MEDIATION IN NLSY97 DATA

- Then run your multilevel models with those variables!

4. sexual.age = sex.agg.c + sex.cwc

5. parental.monitoring = sex.agg.c + sex.cwc

6. sexual.age = sex.agg.c + sex.cwc + parental.monitoring.agg.c + parental.monitoring.cwc

1-1-1 MEDIATION IN SPSS

- ```
  MIXED sexual.age WITH sex.agg.c sex.cwc
  /FIXED= sex.agg.c sex.cwc
  /PRINT= SOLUTION
  /RANDOM=INTERCEPT | SUBJECT( school.id ).
  ```
- ```
  MIXED parental.monitoring WITH sex.agg.c sex.cwc
   /FIXED= sex.agg.c sex.cwc
  /PRINT= SOLUTION
  /RANDOM=INTERCEPT | SUBJECT( school.id ).
  ```
- ```
  MIXED sexual.age WITH sex.agg.c sex.cwc parental.monitoring.agg.c parental.monitoring.cwc
   /FIXED= sex.agg.c sex.cwc parental.monitoring.agg.c parental.monitoring.cwc
  /PRINT= SOLUTION
  /RANDOM=INTERCEPT | SUBJECT( school.id ).
  ```
  ```

1-1-1 MEDIATION IN R

- `lme( sexual.age ~ sex.agg.c + sex.cwc, random=~1|school.id)`
- `lme( parental.monitoring ~ sex.agg.c + sex.cwc, random=~1|school.id)`
- `lme( sexual.age ~ sex.agg.c + sex.cwc + parental.monitoring.agg.c + parental.monitoring.cwc, random=~1|school.id )`

EXAMPLE MEDIATION IN NLSY97 DATA



*$p < .05$, **$p < .01$, ***$p < .001$

SUMMARY: LEVELS IN MULTILEVEL MEDIATION

- If both your predictor and mediator are at Level 2:
  - Do the causal steps approach with MLM like normal
- If either the mediator or predictor are at Level 1:
  - Replace that variable with its group mean *and* its group-mean centered value

Zhang, Zyphur, & Preacher (2009)

APPLICABILITY ACROSS GROUPS

- Key Issue:
  - *Does the mediational model apply equally across all your groups?*
- Depends on:
  - Whether you have random slopes for variables in your mediational model

Kenny, Korchmaros, & Bolger (2003)

NO RANDOM SLOPES

- Do your mediation analysis like I just told you, depending on whether you have a 2-2-1, 2-1-1, or 1-1-1 model
  - That's it, don't worry about anything else
  - Calculate a Sobel test using the estimates and standard errors for each path using an online calculator
- *Note*: This is also cool if *only one* slope in the indirect path is random

Kenny, Korchmaros, & Bolger (2003)

RANDOM SLOPES FOR *BOTH* PATHS *a* AND *b*

- Do your mediation analysis like I just told you, depending on whether you have a 2-2-1, 2-1-1, or 1-1-1 model
- You will also need to compute, test, and report the covariance of these slopes, σ_{ab}
- You will need to manually compute your Sobel *z*

Kenny, Korchmaros, & Bolger (2003)

POPULATION COVARIANCE, σ_{ab}

- *Estimates how reliably your mediational model explains the data across your Level 2 units when both paths of the indirect effect are modelled as random slopes*
- Interpretation of σ_{ab} depends on significance and sign
  - If σ_{ab} is not significantly different from 0, then your mediational model is true across all Level 2 groups
  - If σ_{ab} is significant, then look at the sign (+/-)
    - Positive σ_{ab} indicates that groups with larger a_j paths also have larger b_j paths
    - Negative σ_{ab} indicates that groups with larger a_j paths have smaller b_j paths (and vice versa)

Kenny, Korchmaros, & Bolger (2003)

CALCULATING POPULATION COVARIANCE, σ_{ab}

- Easiest method of doing it:

1. Run the models that are used for the indirect path
2. Save the random slope estimates for a_j and b_j
3. Calculate their covariance and interpret its sign
4. Test the correlation of the estimates of a_j and b_j

SOBEL TEST

- If the relationship between the slopes of *a* and *b* is not significant, you can do a Sobel Test like normal (use an online calculator)

- But, if the relationship between the slopes of *a* and *b* IS significant, then you need to calculate Sobel *z* by "hand" for the significance of the indirect effect:

$$z = \frac{ab}{\sqrt{b^2\sigma_a^2 + a^2\sigma_b^2 + \sigma_a^2\sigma_b^2 + 2ab\sigma_{ab} + \sigma_{ab}^2}}$$

REPORTING YOUR MULTILEVEL MEDIATION

- People will want to know:
  - The type of multilevel mediation (e.g, 2-1-1, 1-1-1, 2-2-1) in English words
  - The specifics of each multilevel model involved (i.e., model specification of fixed and random parameters, covariance matrix, df estimation)
  - The results of each model and of the Sobel Test
  - (If both paths *a* and *b* are random) The value and significance of σ_{ab}

MULTILEVEL MODELING

- What it is
  - *An extension of regression where parameters (i.e., intercept, slopes) are predicted, in addition to predicting the outcome*
- When to use it
  - *Your data is hierarchical in nature; your observations are not independent*
- How many levels?
  - *When the levels are clear-cut, then however many seem appropriate*
  - *When the lowest-level of observation could be classified into one group OR another (i.e., the category lines are not rigid), then you use a cross-classified model*
- Mediation in MLM
  - *When conducting mediation in MLM, you must consider the levels of your predictor and mediator, as well as the consistency of your mediational model across groups*

FURTHER RESOURCES

- Questions, future help, and feedback:
  - elizabeth.page-gould@utsc.utoronto.ca
  - http://page-gould.com/mlm/aps/
- Some good MLM reference books:
  - (SPSS-focused) Bickel, R. (2007). *Multilevel analysis for applied research: It's just regression!* New York, NY, US: Guilford Press.
  - (R-focused) Wright, D. B., & London, K. (2009). *Modern regression techniques using R: A practical guide for students and researchers*. Thousand Oaks, CA, US: Sage Publications.
  - (SAS-focused) Singer, J. D., & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence*. New York, NY, US: Oxford University Press.
  - Raudenbush, S. W., & Bryk, A. S. (2001). *Hierarchical linear models: Application and data analysis methods* (2nd ed). Thousand Oaks, CA, US: Sage Publications.

¡¡THANK YOU!!

- Workshop Sponsors:
  - Association for Psychological Science
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  - Howard Seltman (Carnegie Mellon)
  - Tom Wickens (UC Berkeley)
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  - Ian Dennis Miller
  - Nicholas Rule
  - Ulrich Schimmack
  - Jennifer Tackett
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